INTRODUCTION

Nonlinear circuits are known to have multiple mathematical solutions for the same set of input parameters, but only one is observed in practice; that known as the stable solution, as opposed to non-observable or unstable solutions, which cannot withstand the noise fluctuations affecting physical systems and vanish.

To predict nonlinear circuit behaviour, harmonic balance (HB) is a popular and useful frequency-domain technique. But given its mathematical nature, it can only converge to mathematical solutions being harmonically related to the input sources into the circuit. Nevertheless, the mathematical solutions to a nonlinear system do not necessarily need to be harmonically related to the input signals (i.e., autonomous oscillator or non-harmonic spurious from amplifiers) and here is where the conundrum begins.

The only thing that can be said about a converged result from HB is that it is a steady-state mathematical solution to the nonlinear circuit. Now, on the other hand, there is also time-domain integration for the prediction of nonlinear-circuit behaviour. While the time-domain approach typically converges to observable solutions, it is not always practical for the optimization of nonlinear circuits under a particular steady-state response, as these methods may require long simulation times before reaching the stationary regime. Also, they can be inhospitable to some circuit elements or functions. This is when/where the AG feature being employed within the NI AWR Design Environment™ can be advantageous.
The application note analyzes a very simple LC resonator in time and frequency domains. It goes on to illustrate how oscillatory solutions can be found and reproduced from a nonlinear circuit with the aid of AG. A second application note expands how to apply AG from this simple circuit to the nonlinear circuit simulation of a forced Van der Pol oscillator.

EXAMPLE CIRCUIT: LC RESONATOR
While a pure LC resonator is not strictly speaking a nonlinear circuit, its use as an example is highly effective for illustrating the basic principles of AG within this application note.

TIME-DOMAIN ANALYSIS
A pure LC resonator cannot maintain a stable dc voltage or current. The smallest electrical perturbation will make it oscillate, keeping its energy in a constant conversion cycle between electric and magnetic forms. Without any power loss, the system develops a constant amplitude oscillation.

This can be reproduced in the time domain using AWR’s APLAC® transient simulator within its Microwave Office® design environment. This is done by means of reactive elements with initial condition settings, such as the Cap_AP and/or Ind_AP. Figure 1 shows this circuit (top) and its response (bottom).

Figure 1: Stable oscillation from a lossless LC resonator.
Looking deeper into this circuit, however, it has an unstable dc solution and an infinite number of oscillatory solutions (amplitude/phase states), each being unique for a particular set of initial conditions (loop current and/or node voltage). With the addition of resistive losses, a new circuit results (Figure 2), having a stable dc or static solution, which is always recovered after any small signal perturbation induced by initial values in a loop current and/or node voltage.

**FREQUENCY-DOMAIN ANALYSIS**

Similarly, the above circuits can be analyzed in the frequency domain by connecting them to a small signal current source and performing a frequency sweep. Measuring the small signal voltage at the observing node (in this case the capacitor voltage), an admittance transfer function can be determined. This is effectively an input/output transfer function (input current related to output voltage) and shows the characteristic roots of the system.

As a point of reference, a linear system is said to be unstable when it contains poles in the right hand plane (RHP). The term unstable refers to the dc solution of the system, which is subject to a small signal perturbation. The presence of a couple of complex conjugate poles in RHP is equivalent to having a negative real part of the transfer function at the frequency where the imaginary part crosses zero with positive slope.

Having a solid understanding of circuit theory, the designer then knows that the pure LC resonator contains several complex conjugate poles in the imaginary axis, giving it a critical stability. Consequently, any oscillation produced by a small-signal perturbation will be sustained. The analysis of its admittance transfer function $Y_{tf}$ (Figure 3 - top) corroborates this behaviour and the critical stability is interpreted as the admittance, being purely imaginary (zero real part) at the resonance. On the other hand, the lossy resonator (Figure 3 - bottom) offers a positive real part, showing that steady-state oscillations cannot be sustained. The dc solution from this circuit is stable and thus will be observable.

Real systems are constantly subject to natural noise fluctuations, and this example shows that the pure LC resonator cannot maintain a stable static response and oscillates, however, this circuit is not representative of any physical passive system. According to the Kurokawa condition for starting oscillations from a nonlinear circuit, a negative value of the real part of the small-signal admittance is required to start an oscillation with growing amplitude; negative real parts are only possible if energy

![Figure 2: In a lossy LC resonator, the static or DC solution (0 V) is recovered after any small signal noise perturbation.](image1)

![Figure 3: Stability analysis of the DC solution from the pure LC (top) and lossy resonator (bottom).](image2)

![Frequency sweep](image3)
is applied into the system (part of this energy is transferred to the developing oscillation). Because the relationships between large signal currents and voltages are different from those of the small signal case (and are functions of the amplitude and frequency), the oscillation attains a saturation point by the law of energy conservation and reaches its final spectral parameters, becoming the observable solution. This solution is stable, because of the robust to small-signal perturbations and has no poles in RHP.

The simple stability analysis performed on the dc solution of linear circuits can be applied to determine the stability properties of solutions from nonlinear circuits under large signal operation. To do this however, it is first necessary to stimulate different solutions in a nonlinear circuit and to reproduce them by means of HB simulations. Those solutions are later subject to a small-signal perturbation analysis from which their stability properties can be determined. All these steps require the use of auxiliary generators.

AUXILIARY GENERATOR (AG) CONCEPT

To get started with this illustration, solutions will be stimulated in a circuit (other than the ones obtained from a standard HB simulation) by introducing additional signal generators (AGs) that can act only as catalysts. It is important to note that the AGs are not part of the circuit.

This is accomplished by imposing a non-perturbation condition on those sources; that is, no current is allowed to flow between the source and the circuit. This is equivalent to having the source disconnected from the circuit; henceforth, these sources will be referred to as AGs. Figure 4 shows the result obtained (bottom) for the lossless LC resonator when an AG is introduced into the circuit schematic (top).

In the case of Figure 4, the AG voltage has been set to a fix value of 1V (a linear circuit never saturates!). When simulating nonlinear circuits, the AG voltage and frequency can both be optimized to find a particular oscillatory solution satisfying the non-perturbation condition.

This condition is expressed in terms of the admittance “seen” from the AG: \( Y_{\text{target}} = I_{\text{AG}} / V_{\text{AG}} = 0 + j \cdot 0 \) at the AG frequency. Imposing the zero complex value to the admittance and not just to the AG current, \( I_{\text{AG}} \), prevents the optimizer from converging towards the trivial 0V solution for the AG voltage, \( V_{\text{AG}} \). After performing an optimization on the AG frequency, \( f_{\text{AG}} \), we obtain very low values for the admittance and AG current (Table 1). They are considered to be small enough for a good solution.

<table>
<thead>
<tr>
<th>Freq. (GHz)</th>
<th>Re( (Y_s) )</th>
<th>Im( (Y_s) )</th>
<th>mag( (I_{\text{AG}}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5915</td>
<td>4.44e-024</td>
<td>7.2513e-008</td>
<td>7.2513e-005</td>
</tr>
</tbody>
</table>

Table 1
When analyzing a nonlinear circuit, the solution will inevitably contain harmonics. In order to satisfy the non-perturbation equation at those frequencies, an ideal filter must be connected between the AG and the circuit. This prevents the one-frequency AG source short-circuiting the solution harmonics at the connecting node.

The APLAC element Zblock_AP is very convenient for this purpose and can be configured to act as an ideal filter. The simulation gives the same results with and without filter because there is only one harmonic in the solution. Note: When using Zblock_AP, the Vharm measurement must be performed with the APLAC-HB simulator.

**AG WITHIN THE NI AWR DESIGN ENVIRONMENT**

AG can be implemented within the NI AWR Design Environment by invoking the APLAC frequency-dependent impedance element Zblock_AP. This element is defined as $\text{TONE} = N \times H_1 \times X_1 \ldots H_N \times X_N$, where $N$ is the number of harmonics $H_1$ to $H_N$, at which the impedances $X_1$ to $X_N$ are specified. $X_i$ can be real or complex (re, im).

In single-tone HB analysis, $H_i$ is an integer multiple of the fundamental frequency $f_1$, whereas in multi-tone HB, $H_i$ is a vector $[m,n]$ or $[m,n,k]$ for 2-tone and 3-tone HB analysis respectively.

Zblock_AP can be seen as the parallel connection of $X_i$ impedances, each having its value at only the exact frequency $f = m \cdot f_1 + n \cdot f_2 + k \cdot f_3$, and being infinite at any other frequency. This element is thus an open circuit at frequencies other than those specified by the tones $H_1 \ldots H_N$.

For the AG configuration, $N=1$ and $X_1 = 0$ have been chosen (refer to Figure 5). An NCONN element that has been named “Vnode” is used for the connection of the AG to the circuit.

![Figure 5: Two implementations of the auxiliary generator: (a) single-tone HB analysis with $f_{AG} = f_1$ (fundamental), and (b) two-tone HB analysis with $f_{AG} = f_2$ (second tone). The I_METER is an optional element as the admittance can also be simulated using the large signal admittance (Ycomp) measurement in the NI AWR Design Environment.](image)

An AG used in circuits with no signal generators (autonomous or free running) is similar to the OSCAPROBE element from the Microwave Office element catalog. The same results are obtained with both techniques. (NOTE: the AG phase in this case is set to zero because the phase reference is irrelevant in autonomous circuits.)
In this particular example, the OSCAPROBE analysis does not give a satisfactory result due to the absence of a nonlinear element in the LC resonator to help the probe determine the amplitude of the searched solution. The simulation result from the HB engine within the NI AWR Design Environment is not ideal: It does find the correct value of the oscillating frequency, it gives a non-physical amplitude voltage of 0.59 V (Figure 6).

Turning now to the analysis of the lossy LC resonator with the AG included, it too produces no satisfactory solutions (no convergence for the non-perturbation condition is reached). This is because this circuit has no oscillatory solution. The steady state 0V can not be found with the AG because the non-perturbation equation prevents the AG voltage convergence towards a zero value.

As in the previous case, the simulation of this circuit with OSCAPROBE produces a less than satisfactory result (Figure 7). The analysis with APLAC-HB indicates that no convergence to a solution has been reached. This is in agreement with the AG result.

The AG technique provides the user with more flexibility than the OSCAPROBE – as any of the three AG parameters (amplitude, frequency, and phase) can be controlled – and it is also compatible with the presence of other independent signal generators in the circuit and not restricted solely to autonomous circuits.

Finally, an HB simulation (Figure 8) is performed on the LC resonator. The APLAC-HB solver correctly identifies the dc solution, but it is the same for the resistive and reactive configurations, showing that the HB analysis is insensitive to the stability properties of the converged solutions.
CONCLUSION

While harmonic-balance and time-domain simulation techniques are often tried-and-true methods for nonlinear analysis, they can be inhospitable to some circuit elements or functions. This is when/where the AG feature being employed within NI AWR Design Environment can be advantageous.

This application note showed how the AG technique can be used to reproduce results with a harmonic balance simulator that were in principle reserved to transient and small signal analysis because of the nature of the circuit under study; a passive linear LC resonator with no signal sources. AG offers an increased flexibility to the designer in cases where signal sources are present but a new solution –not harmonically related to these sources- is to be studied.

REFERENCES


This application note introduces the technique known as auxiliary generator (AG). It is very useful for broadening the set of solutions that can be reproduced from a harmonic balance (HB) analysis and can help study their stability properties in order to optimize a nonlinear circuit for a desired response, while at the same time avoiding undesired modes such as unwanted oscillations. It is of particular interest for the analysis and design of all kinds of nonlinear circuits, including–but not restricted to–oscillators and power amplifiers.

AWR would like to thank José Luis Flores, Microwave & RF Engineer, for his contributions to this application note.

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